Cours ware for pre-service teachers

<u>Developers</u>

Tyali, Zimasile

Majiyezi, Phumlani

Ntanjana, Nonkosikho

SECTION A: PRELIMINRY DISCUSION

UNIT 13: TRIGONOMETRY

Topic overview

- This topic is the third of five topics in Term 3.
- This topic runs for two weeks (9 hours).
- It is presented over seven lessons.
- The lessons have been divided according to sub-topics
- Trigonometry counts 33% of the final Paper 2 examination.

Learner outcomes/Objectives for grade 11

- Prove and apply the sine, cosine, and area rules
- Solve problems in two dimensions using the sine, cosine, and area rules.

Learner outcomes/Objectives for grade 12

- Solve problems in 3 dimensions
- Problems can include compound or double angles.

THE NSC DIAGNOSTIC

According to NSC Diagnostic Reports there are several issues pertaining to solving triangles using Trigonometry.

These include:

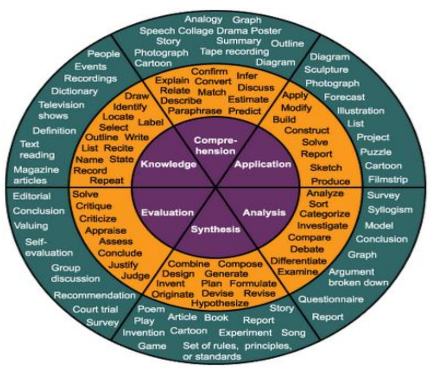
- recognizing right-angled triangles and applying the Theorem of Pythagoras

- Difficulty in selecting the sides/angles required. Many fail to see that two triangles share a side or an angle
- Difficulty in seeing which rule was required z poor algebraic manipulation skills (changing the subject of the formula).

ASSESSMENT OF THE TOPIC

- Two tests, with memorandum, are provided in the Resource pack. The tests are aligned to CAPS in every respect, including the four cognitive levels as required by CAPS (page 53).
- The questions usually take the form of solving triangles, sometimes related to real-life situations. Proofs could also be asked.
- Monitor each learner's progress to assess (informally) their grasp of the concepts. This information can form the basis of feedback to the learners and will provide you valuable information regarding support and interventions required.

Bloom Taxonomy wheel



- **The selection of questions will be tested by bloom taxonomy cognitive levels.**

SECTION B: OUTCOMES

1. Pedagogical stance on the teaching of trigonometry: concepts and applications

2. Conceptual, factual and procedural development of trigonometry: this unit is being intended to support you in being able to develop more sophisticated skills and techniques for teaching,

- Trigonometric ratios revision
- Trigonometric identities
- Compound angles
- Double angles
- Trigonometric equations
- Trigonometric functions

3. Inductive and deductive reasoning
4. Blooms taxonomy
5. Further practical examples and solutions
6. Technology of teaching trigonometry (the use of GEOGEBRA APP)

SECTION C: conceptual, factual, procedural developments and discussions

Activities for sessions

1. **TEXTBOOKS**
   The provided below links are for the textbooks that we used to construct this work, for grade 10 https://www.siyavula.com/read/maths/grade-10/trigonometry-part-1, 11 https://www.siyavula.com/read/maths/grade-11/trigonometry,
   and 12 https://www.siyavula.com/read/maths/grade-12/trigonometry.

2. **DOUBLE ANGLES**

   The provided link will give a learner an introduction to double angle identities, https://www.bing.com/videos/search?q=trigonometry+double+angles&qpvt=trigonometry+double+angles&view=detail&mid=9B2904774227C7BFFF9A9B2904774227C7BFFF9A&&FORM=VRDGAR, then if a learner wants to explore in this topic, examples are in the following link:
   https://www.bing.com/videos/search?q=trigonometry+double+angles&qpvt=trigonometry+double+angles&view=detail&mid=7FCF1D11E1A0C90C994C7FCF1D11E1A0C90C994C&&FORM=VRDGAR&ru=%2Fvideos%2Fsearch%3Fq%3Dtrigonometry%2Bdouble%2Bangles%26qpvt%3Dtrigonometry%2Bdouble%2Bangles%26FORM%3DVDRE, to explore more on this topic the following link gives solution on trig equation about double angles:
   https://www.youtube.com/watch?v=yFWlxK4_AKE

3. **COMPOUND ANGLES**

   The provided link will give a learner, techniques of expanding and not expanding double angles https://www.youtube.com/watch?v=rm5SVUJA2Bw this one will give a background on double angle trig identities and double angle trig calculus https://www.youtube.com/watch?v=sePxgAgrin4

4. **TRIGONOMETRIC IDENTITIES**
   The provided links will give a learner an introduction to ways of proving trigonometric identities https://www.youtube.com/watch?v=Xpo_-RQtMhs and https://www.youtube.com/watch?v=UIOu4BBwIps and this one focuses on verifying trig identities with hard examples including fractions https://www.youtube.com/watch?v=Rf05H8ogHLg

5. **TRIGONOMETRIC EQUATIONS**
   The provided link will give a learner strategies of working out angles in a non-right angled triangles https://www.youtube.com/watch?v=bDPRWJdVzfs
   This following link will focus on grade 11 trig equations
   https://youtu.be/XwQNKg0cZVs?t=15
   The last link show how to solve trig equations involving primary ratio (sine, cosine and tangent) https://www.youtube.com/watch?v=2usVyTSYDyw.

6. **ARTICLES ON TRIGONETRY FET**

   A guide on the misconceptions and how the trigonometry subject has been taught and the outcomes, check the following article,
   https://www.researchgate.net/publication/267851364_Trigonometry_Learning
   and
   https://www.researchgate.net/publication/228451317_Teaching_and_learning_trigonometry_with_technology.

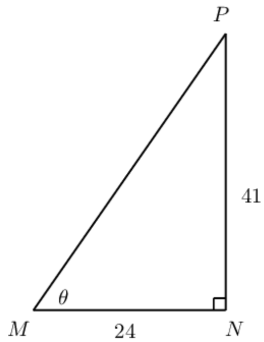
OTHER TECHNOLOGIES THAT CAN BE USED

- GEOGEBRA
- EQUATION EDITOR (MICROSOFT WORD)
- CALCULATOR

NONKOSIKHO

1. TRIGONOMETRIC RATIOS REVISION

- Calculate the value of θ in the right-angled triangle MNPMNP (correct to one decimal place):



ANSWER: taking into account that this is a right angled triangle so Pythagoras theorem may be applied and identifying the opposite and adjacent sides and the hypotenuse.

tanθ= $\frac{opposite\ side}{adjacent\ side}$ = $\frac{41}{24}$

tan−1($\frac{41}{24}$)=59,7°

2. TRIGONOMETRIC IDENTITIES

Simplify the following using trigonometric identities

- tan2θ×cos2θtan2θ×cos2θ

ANSWER : An identity is a mathematical statement that equates one quantity with another. Trigonometric identities allow us to simplify a given expression so that it contains sine and cosine ratios only. This enables us to solve equations and also to prove other identities.

$\frac{sin2\theta}{cos2\theta}$ × cos2θ × $\frac{sin2\theta}{cos2\theta}$ × cos2θ

= Sin2θ × cos2θ

3. COMPOUND ANGLES (GRADE 12)

$$\cos(A-B) = cosA.cosB + sinA.SinB$$

$$\cos(A+B) = cosA.cosB - sinA.SinB$$

$$\sin(A+B) = sinA.cosB + cosA.sinB$$

$$\sin(A-B) = sinA.cosB - cosA.sinB$$

EXAMPLE 1. Simplify the following using the compound angles

- $cos\ cos\ (A+60°)$ $+cos\ cos\ (A-60°)$

Answer: cosAcos60º-sinAsin60º – cosAcos60º

=2cosAcos60º

= 2cosA($\frac{1}{2}$)

= cosA

4. DOUBLE ANGLES (GRADE 12)

1. $\cos 2A = \cos^2 A - \sin^2 A \rightarrow$ A is half the original angle

   $= 2\cos^2 A - 1$

   $= 1 - 2\sin^2 A$

2. $\sin 2A = 2\sin A \cos A \rightarrow$ A is half the original angle

Proof 1.

Show that $\cos\ 2A = \cos^2 A - \sin^2 A$ using double angles.

ANSWER:

$\cos\ (A + B)$ =cosAcosB + sinAsinB

cos(A+A)= cosAcosA - sinAsinA = $\cos^2$A-$\sin^2$A

5. TRIGONOMETRIC EQUATIONS (GRADE 12)

Determine the general solution $2\cos^2\theta + \cos\cos\theta - 1 = 0$

ANSWER: (2cosθ-1)(cosθ+1)=0

Cosθ=$\frac{1}{2}$ or cos$\theta = -1$

RA(reference angle)=0º or RA=60º

THEREFORE θ = 60º+360ºk or θ= 180º+ 360ºk and k belongs to a set of integers

6. TRIGONOMETRIC FUNCTIONS

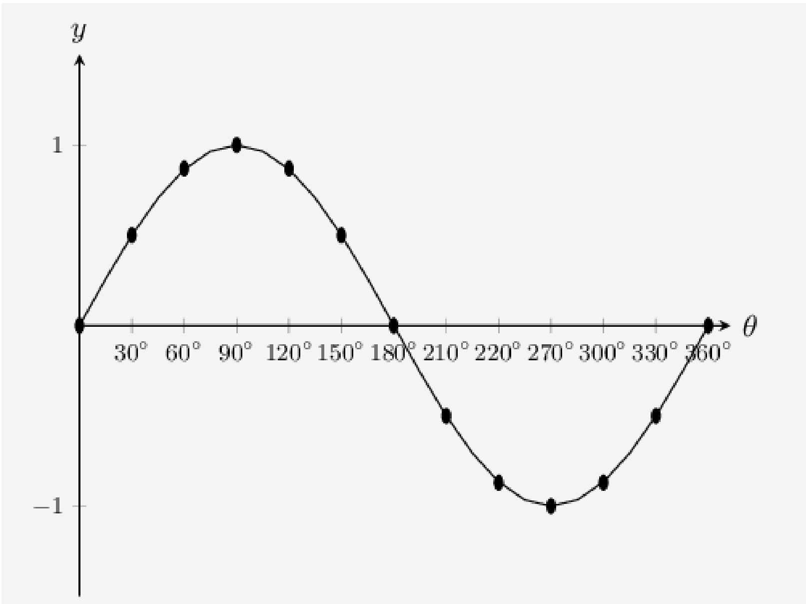
GRAPH 1. THE SINE FUNCTION: $y = a\sin\sin b(x + p) + q$

Sketch the graph of $y = \sin\sin x\ for\ x \in [-360°; 360°]$

- Make use of a table or a calculator to determine the critical points on the graph

- The endpoints of the domain must be indicated. i.e. $x = -360°\ and\ x = 360°$.
- All the intercepts with the x and y axis must be indicated as well as all minimum and maximum points (turning point).

Answer

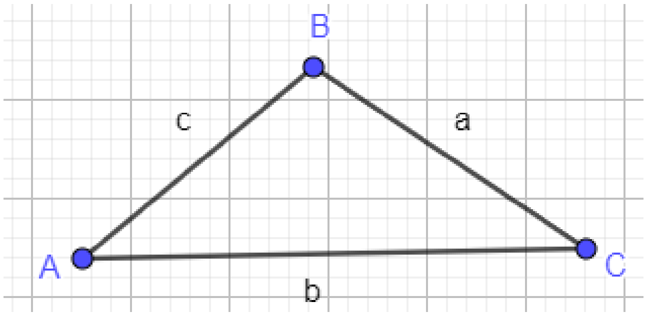


		$y = sinx$
1	Maximum Value	1, at $x = 90°$
	Minimum Value	-1, at $x = 270°$
2	Domain	$x \in [0°; 360°], x \in R$
	Range	[-1;1], $y \in R$
3	x-intercept	$0°, 180°, and\ 360°$
4	Amplitude	1
5	Period	$360°$

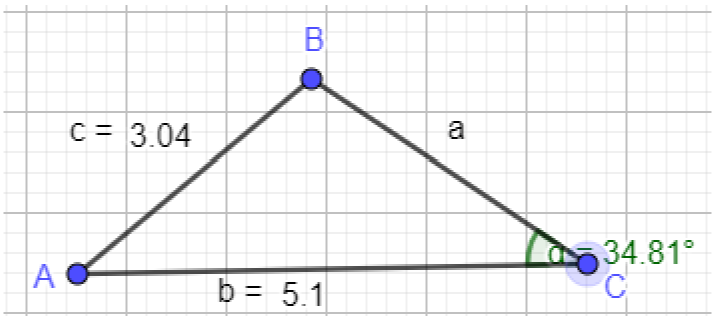
A

7. TRIGONOMETRY (SINE, COSINE AND AREA RULES)

SINE RULE:



: $\frac{SinA}{a} = \frac{SinB}{b} = \frac{SinC}{c}$ or $\frac{a}{sinA} = \frac{b}{sinB} = \frac{c}{sinC}$



- You apply the **Sine rule** if you are given the values of?
- Use a given triangle below to find the unknown sides and angle.

Rubric

	1	2	3	4
PROBLEM SOLVING	Little or no understanding of the problem in evidence	Numerous errors when solving problems	Few errors when solving problems	No errors when solving problems
TRIGNOMETRY CONTENT	Demonstrate little or no knowledge or application of maths skills	Demonstrate a limited knowledge	Demonstrate a general knowledge	Accurately com

		and application of maths skills	and application of maths skills	muni cates soluti ons to probl ems and conc epts
MATHEMATICS COMMUNICATI ON	Inaccurately communicates solutions to problems and concepts	Limited communicatio n of solutions to problems and concepts	Satisfactorily communicate s solutions to problems and concepts	Accu rately com muni cates soluti ons to probl ems and conc epts
PRESENTATIO N	The presenter is unable to follow the steps taken in the solution.	The solutions are difficult to follow at times	Solutions presented in a logical manner	Soluti ons prese nted in an easy follo w step-by-step mode l
USE OF MATHEMATICA	No mathematical technology is used or attempted	Some mathematical	Mathematical technology	Math emati

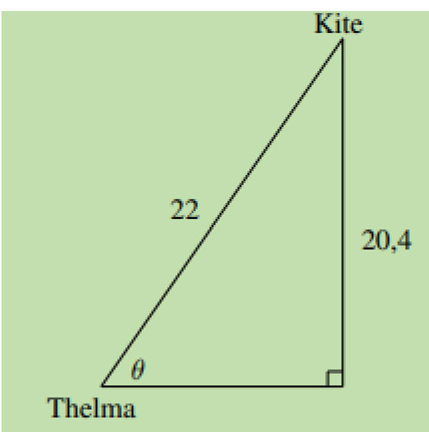
L TECHNOLOGY		technology is presented but not correctly used	correctly used	cal technology is used correctly and accurately.

PHUMLANI

8. TRIGONOMETRIC RATIOS

Thelma flies a kite on a 22 m piece of string and the height of the kite above the ground is 20,4 m. Determine the angle of inclination of the string (correct to one decimal place).

Answer:



$$sin\ sin\ \theta\ = \frac{opposite}{hypotenuse}$$

$$= \frac{20,4}{22}$$

= (0,927 . . .)

∴ θ = 68, 0°

Analysis

Blooms taxonomy

This is an example of a question that tests knowledge/ comprehension.

Here students recall what has been taught about Trigonometric ratios.

9. TRIGONOMETRIC IDENTITIES

- Simplify the following using trigonometric identities

  $\frac{1}{cos2\theta} - \tan 2\theta$

Answer

$$\frac{1}{cos^2\theta} - tan^2\theta = \frac{1}{cos^2\theta} - \left(\frac{sin\theta}{cos\theta}\right)^2$$

$$= \frac{1}{cos^2\theta} - \frac{sin^2\theta}{cos^2\theta}$$

$$= \frac{1 - sin^2\theta}{cos^2\theta}$$

$$= \frac{cos^2\theta}{cos^2\theta}$$

= 1

Analysis

Here a student must **apply** the trigonometric identities, based similar examples from class.

Of course, if this problem has been covered well enough for the student to be able to produce it by rote, answering it correctly simply requires recall.

10. COMPOUND ANGLES (GRADE 12)

$\cos(A - B) = cosA.cosB + sinA.SinB$

$\cos(A + B) = cosA.cosB - sinA.SinB$

$\sin(A + B) = sinA.cosB + cosA.sinB$

$\sin(A - B) = sinA.cosB - cosA.sinB$

Prove the following using the compound angles.

- $sin\ sin\ (90° - \theta) = cos\ cos\ \theta$

Answer

$sin\ sin\ (90° - \theta) = sin90°.cos\theta - cos90°.sin\theta$
$= 1.cos\theta - 0$
$= cos\theta$

Analysis

In this question the term **prove** is under **apply** according to blooms taxonomy is second cognitive level.

In this question, the student must apply what has been taught about compound angles to perform such questions.

11. DOUBLE ANGLES (GRADE 12)

1. $\cos 2A = \cos^2 A - \sin^2 A \rightarrow$ A is half the original angle

$= 2\cos^2 A - 1$

$= 1 - 2\sin^2 A$

2. $\sin 2A = 2\sin A \cos A \rightarrow$ A is half the original angle

Proof 2.

Express cos2A in terms of $cos\ cos\ A$ and $sin\ sin\ A$ only.

Answer

$cos2A = cos^2A - sin^2A$

$= cos^2A - (1 - cos^2A)$

$= 2cos^2A - 1$

Or

$cos2A = cos^2A - sin^2A$

$= (1 - sin^2A) - sin^2A$

$= -2sin^2A + 1$

Analysis

Looking the question there's a term **express** of which is under **analyzing** according to blooms taxonomy third cognitive level.

Here the student is expected to know how to analyze the double angles before they can attempt the question.

12. TRIGONOMETRIC EQUATIONS (GRADE 12)

Solve for x if 2$sin\, sin\, x\ \ cos\, cos\, x\ \ = sin\, sin\, x$

Answer

$x\ \ cos\, cos\, x\ \ = sin\, sin\, x$

$sin\, sin\, x\ \ cos\, cos\, x\ \ - sinx = 0$

$sinx(cosx - 1) = 0$

$sinx = 0\ or\ cosx = \frac{1}{2}$

$\therefore RA = 0°\ OR\ RA = 60°$

$\therefore x = 0° + 360°k\ \ or\ \ x = 180° - 0° + 360k\ \ or\ \ x = \pm 60° + 360°k; k \in Z$

Analysis

In this question there's a term **solve** falls under **apply** it's a second cognitive level according to blooms taxonomy

Here the student are expected to apply the previous understanding about trigonometric equation to be able to solve the given question.

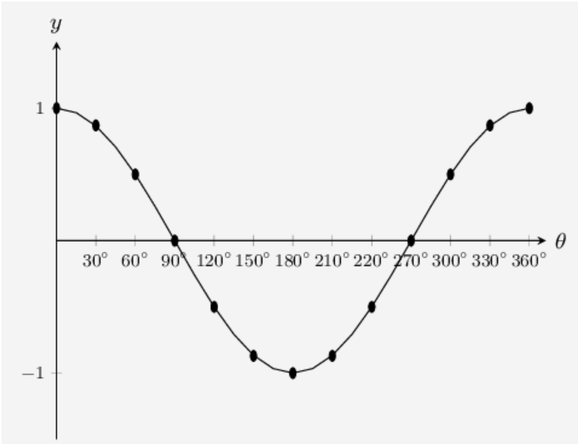
13. TRIGONOMETRIC FUCTIONS

GRAPH 2. THE COSINE FUNCTION: $y = a\ cos\ cos\ b(x + p) + q$

Sketch the graph of $y = cos\ cos\ x\ for\ x \in [0°; 360°]$

- Make use of a table or a calculator to determine the critical points on the graph
- The endpoints of the domain must be indicated. i.e. $x = 0°\ and\ x = 360°$.
- All the intercepts with the x and y axis must be indicated as well as all minimum and maximum points (turning point)

Answer



		$y = cosx$
1	Maximum Value	1, at $x = 0°$ and $360°$
	Minimum Value	-1, at $x = 180°$
2	Domain	$x \in [0°; 360°], x \in R$
	Range	[-1;1], $y \in R$
3	x-intercept	$90°$ and $270°$
4	Amplitude	1
5	Period	$360°$

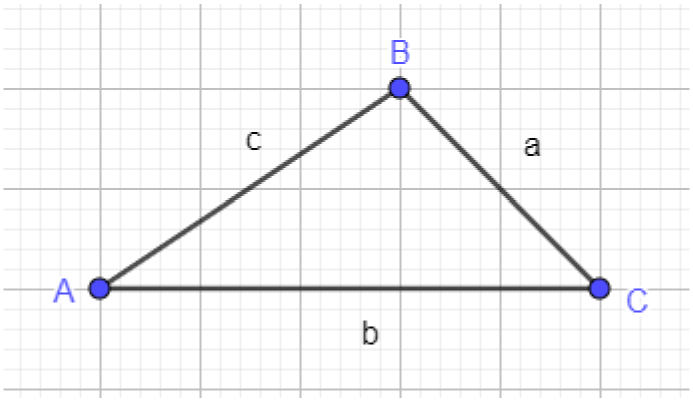
Analysis

In this question there is a term **determine** is under **apply** (second cognitive level) according to blooms taxonomy.

Here the students they are required to apply what they know about cosine functions in order for them to answer more questions based on functions.

14. TRIGONOMETRY (SINE, COSINE AND AREA RULES)

COSINE RULE:

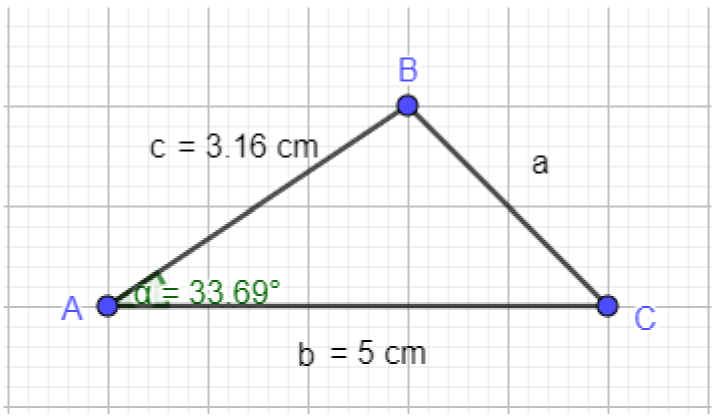


$a^2 = b^2 + c^2 - 2bc \cos\cos A$

$b^2 = a^2 + c^2 - 2ac \cos\cos B$

$c^2 = a^2 + b^2 - 2ab \cos\cos C$

- We apply the **Cosine rule** if you are given the values of?
- Use a triangle given below to find the unknown sides and angles.



Answer

$a^2 = b^2 + c^2 - 2bcCosA$

$a^2 = (5)^2 + (3.16)^2 - 2(5)(3.16)Cos(33.69°)$

$a = \sqrt{5^2 + 3.16^2 - 2(5)(3.16)Cos(33.69°)}$

$a = 2.95\ cm$

Analysis

In this question there's a term **find** that falls under **remembering** the lower cognitive level.

Here the students needs to recall the formulas of a cosine rule first and know that if you're given three sides or two side and included angle in a triangle we need to use a cosine rule to perform that particular question.

Mathematics Rubric for Presentations

Students are rated on criteria in the following three broad categories:

- Mathematical Content
- Presentation Style
- Clarity and Organization

Using the following rubric marks:

- 3 – Criteria fully met
- 2 – Criteria mostly met
- 1 – Criteria minimally met
- 0 – Criteria not met

Brief comments may be added for each category noting particular strengths or weaknesses of the presentation/presenter in that category.

Details on Categories and Criteria:

I. Mathematical Content

- Content presented is mathematically accurate
- Demonstrates adequate understanding of content and is able to answer questions related to the content
- Content is appropriate to the assignment/class
- Level of sophistication of the mathematics is appropriate to the class
- Appropriate amount of content is presented

II. Presentation Style

- Voice is of appropriate volume and is clear
- Pace is not too fast or slow

- Appropriate use of technology
- Sufficient preparation and practice evident in presentation
- Presenter engages appropriately with audience

III. Clarity and Organization

- There is a clear overall organization to the presentation
- Sufficient and clear examples are given when appropriate
- Sufficient motivation for the mathematics is given when appropriate
- Clear explanations of terminology, theorems, and proofs when appropriate

Sample Rubric Sheet for Presentations in Mathematics

Mathematical Content 3 2× 1 0

Comments: Demonstrates adequate understanding of content and is able to answer questions related to the content

Presentation Style 3 2x 1 0

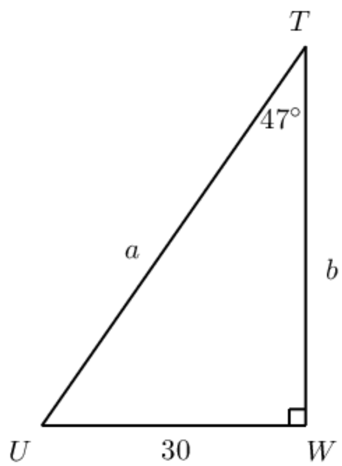
Comments: Voice is of appropriate volume and is clear, Pace is not too fast or slow

Clarity and Organization 3 2 1x 0

Comments: There is a clear overall organization to the presentation, Clear explanations of terminology, theorems, and proofs when appropriate

<u>ZIMASILE</u>

15. TRIGONOMETRIC RATIOS
    - Determine the values of a and b in the right-angled triangle TUW (correct to one decimal place):



ANSWERS:

$$\sin \sin \theta \ = \frac{opposite}{hypotenuse}$$

$$\sin \sin 47° \ = \frac{30}{a}$$

$$a = \frac{30}{\sin \sin 47°}$$

$$a = 41.0$$

Analysis:

Bloom taxonomy:

- Looking at this question the term ''**determine''** is under ''**apply''** (second cognitive level) looking at the verbs in Blooms Taxonomy.
- Which means that, learners are expected to be knowledgeable about trigonometric ratios first before they can answer this question.
- In answering the question, **APPLYING** is the next cognitive level expected.

Deductive reasoning:

- **General procedure is provided such that learner may use to reach a specific conclusion for instance the learners are provided with general rules of trig ratios (**$\sin \sin \theta = \frac{opposite}{hypotenuse}$**) , by using them exact conclusion is expected.**

Inductive reasoning:

- **Would have been using the example (which is in a form of a question in this case) to come up with the general rule, which is moving from specific examples to general conclusion.**

- **Using this reasoning the question was going to be; if $side\ a = 41.0$ and the side b is 30, sides of a triangle what would be a possible rule to get the angle θ?**

Links : [Trigonometry Introduction (Grade 11 Maths) - YouTube]

16. TRIGONOMETRIC IDENTITIES

- Prove the following using trigonometric identities

$\frac{1-sinsin\ \alpha}{coscos\ \alpha} = \frac{coscos\ \alpha}{1+sinsin\ \alpha}$

Answer:

$$LHS = \frac{1 - sin\ sin\ \alpha}{COS\alpha}$$

$$LHS = \frac{1 - sin\ sin\ \alpha}{COS\alpha} \times \frac{1 + sin\ sin\ \alpha}{1 + sin\ sin\ \alpha}$$

$$= \frac{1 - sin\ sin\ \alpha}{COS\alpha} \times \frac{1 + sin\ sin\ \alpha}{1 + sin\ sin\ \alpha}$$

$$= \frac{1 - sin^2\alpha}{cos\ cos\ \alpha(1 + sin\ sin\ \alpha)}$$

$$= \frac{1 - sin^2\alpha}{cos\ cos\ \alpha(1 + sin\ sin\ \alpha)}$$

$$LHS = \frac{cos\ cos\ \alpha}{1 + sin\ sin\ \alpha)}$$

$$LHS = RHS$$

Analysis:

Bloom taxonomy:

- Looking at this question the term ‘‘**prove’’** is under ‘‘**evaluate’’** (second last cognitive level) looking at the verbs in Blooms Taxonomy.
- Which means that, learners are expected to master all least cognitive levels, which is remembering, understanding, applying, and analyze to be able to evaluate using trigonometric identity.
- In answering the question, **evaluation** is the cognitive level expected.

Inductive reasoning:

- **the precise example (which is in a form of a question in this case) to come up with the general rule, which is moving from specific examples to general conclusion, which is not proving the question only but validates the general rules of trig identities.**

Deductive reasoning:

- **if this was used, it would have been General procedure that is provided such that learner may use to reach a specific conclusion for instance the learners are provided with general rules of trig identities such as ($sin^2\theta + cos^2\theta = 1$), by using them exact conclusion is expected.**

Links : Grade 12 Trig - Topic 5 - Trig Identities - YouTube

COMPOUND ANGLES (GRADE 12)

$\cos(A - B) = cosA.cosB + sinA.SinB$

$\cos(A + B) = cosA.cosB - sinA.SinB$

$\sin(A + B) = sinA.cosB + cosA.sinB$

$\sin(A - B) = sinA.cosB - cosA.sinB$

- Simplify the following using the compound angles
  $sin\ sin\ (169°)\ sin\ sin\ (41°) + sin\ sin\ (79°)\ sin\ sin\ (131°)$

Answer:

$sin\ sin\ (180° - 11°)\ \ sin\ sin\ (41°)\ \ + sin\ sin\ (79°)\ sin\ (180° - 49°)$

$sin\ sin\ (11°)\ \ sin\ sin\ (41°)\ \ + sin\ sin\ (79°)\ sin\ (49°)$

$sin\ sin\ (90° - 79°)\ \ sin\ sin\ (90° - 49°)\ \ + sin\ sin\ (79°)\ sin\ (49°)$

$cos\ cos\ (79°)\ \ cos\ cos\ (49°)\ \ + sin\ sin\ (79°)\ sin\ (49°)$

$cos\ (79° - 49°)$

$cos\ (30°)$

$= \frac{\sqrt{3}}{2}$

Analysis:

Bloom taxonomy:

- Looking at this question the term ''**simplify''** is under ''**evaluate''** (one of the top cognitive levels) looking at the verbs in Blooms Taxonomy.

- Which means that, learners are expected to **remember** all compound angles, **understand** them**, apply**, **analyze** in order to be able to **evaluate** in this case.
- In answering the question, **simplification** is closely related to **evaluation**, it is the next cognitive level expected.

Deductive reasoning:

General procedure is provided such that learner may use to reach a specific conclusion for instance the learners are provided with general rules of compound angle ($sin\ sin\ (A-B) = sin\ sin\ A\ \ cos\ cos\ B\ \ -cos\ cos\ A\ \ sin\ sin\ B$ **, by using them exact conclusion is expected.**

Inductive reasoning:

Would have been using the example (which is in a form of a question in this case) to come up with the general rule, which is moving from specific examples to general conclusion.

Link : Show 7: Trigonometry: Compound And Double Angles - Whole Show (English) - YouTube

17. DOUBLE ANGLES (GRADE 12)

1. $\cos 2A = \cos^2 A - \sin^2 A \rightarrow$ A is half the original angle

$= 2\cos^2 A - 1$

$= 1 - 2\sin^2 A$

2. $\sin 2A = 2\sin A \cos A \rightarrow$ A is half the original angle

Proof 3.

- Show that $sin\ sin\ 2A = 2\ sin\ sin\ A\ \ cos\ cos\ A$ using double angles.

Answer

$sin2A = sin\ (A + A)$

$= sinAcosA + sinAcosA$

$= 2sinAcosA$

Analysis:

Bloom taxonomy:

- Looking at this question the term ''**show''** is under ''**remember''** (first cognitive level) looking at the verbs in Blooms Taxonomy.
- Which means that, learners are expected to recall the concept of double angles because the trick of tackling this question recalling basic double angles and use them.
- So, answering the question, **remembering** is the cognitive level expected.

Deductive reasoning:

- **General procedure is provided such that learner may use to reach a specific conclusion for instance the learners are provided with general rules of double angles (**$sin\, sin\, (A - B) = sin\, sin\, A\;\; cos\, cos\, B\;\; -cos\, cos\, A\;\; sin\, sin\, B$ **, by using them exact conclusion is expected.**

Inductive reasoning:

- **Would have been using the example (which is in a form of a question in this case) to come up with the general rule, which is moving from specific examples to general conclusion.**

Link: [Show 7: Trigonometry: Compound And Double Angles - Whole Show (English) - YouTube]()

18. TRIGONOMETRIC EQUATIONS (GRADE 12)

- Determine the general solution of $sin\, sin\, 2x = cos\, cos\, (x - 10°)$

Answer :

$\therefore cos\, cos\, (90° - 2x)\;\; = cos\, (x - 10°)$

$\therefore\; RA = x - 10° \rightarrow 90° - 2x$

$\therefore\; 90° - 2x = x - 10° + 360°k$ OR $90° - 2x = -(x - 10) + 360°k, k \in Z$

$\therefore\; 3x = -100° + 360k$ OR $-2x = -x + 10° - 90° + 360°k$

$\therefore\; 33.33° - 120°k$ OR $-x = -80° + 360°k$

$x = 80° - 360°$

Analysis:

Bloom taxonomy:

- Looking at this question the term ''**determine''** is under ''**apply''** (second cognitive level) looking at the verbs in Blooms Taxonomy.

- Which means that, learners are expected to be knowledgeable about trig identities and double angles first before they can answer this question.
- In answering the question, **APPLYING** is the next cognitive level expected.

Inductive reasoning:

- **the precise example (which is in a form of a question in this case) to come up with the general rule, which is moving from specific examples to general conclusion, which is not proving the question only but validates the general rules of trig identities.**

Deductive reasoning:

- **if this was used, it would have been General procedure that is provided such that learner may use to reach a specific conclusion for instance the learners are provided**
- **with general rules of double angle such as (**$sin\ sin\ (A+B) = sin\ sin\ A\ cos\ cos\ B\ + cos\ cos\ A\ \ sin\ sin\ B\ $**), by using them exact conclusion is expected.**

Link : Grade 12 Trig Equations with Double Angles - YouTube

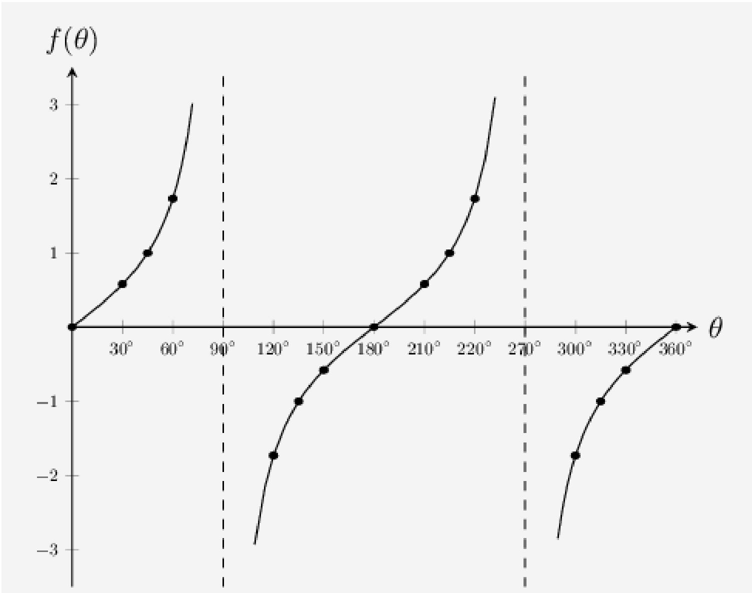
19. TRIGONOMETRIC FUCTION

GRAPH 3. THE TANGENT FUNCTION: $y = a\ tan\ tan\ b(x+p)+q$

Sketch the graph of $y = tan\ tan\ x\ for\ x \in [180°; 180°]$

- All intercepts with the x and y axis must be indicated.
- The endpoints of the domain must be indicated i.e. $x = -180°\ and\ x = 360°$.
- The equations of the asymptotes must be written on the graph.

Answer :



		$y = tanx$
1	Domain	$x \in [0°; 360°], x \in R$
2	Range	[-∞; ∞], $y \in R$
3	x-intercept	$0°, 180°, and\ 360°$
4	Asymptotes	$x = 90°\ and\ x = 270°$
5	Period	$180°$

Analysis:

Bloom taxonomy:

- Looking at this question the term ‘’**sketch’’** is under ‘’**apply’’** (second cognitive level) looking at the verbs in Blooms Taxonomy.
- Which means that, learners are expected to be knowledgeable about trig identities and double angles, and all fundamentals on drawing graphs such as the list in the above table first before they can answer this question.
- In answering the question, **APPLYING** is the next cognitive level expected.

Inductive reasoning:

- **the precise example (which is in a form of a question in this case) to come up with the general rule, which is moving from specific**

examples to general conclusion, which is not proving the question only but validates the general rules of tangent graphs.

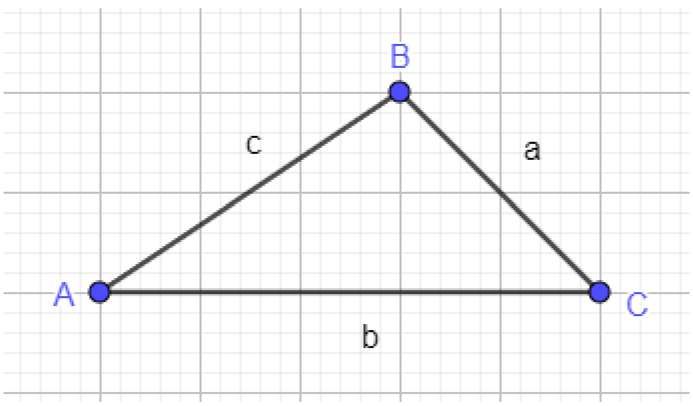
Deductive reasoning:

Links: [Trigonometry Tan Graph - YouTube]

- **if this was used, it would have been General procedure that is provided such that learner may use to reach a specific conclusion for instance the learners are provided with general rules of trig functions such as ($tan\,tan\,\theta$), by using them exact conclusion is expected.**
- **Note, clear explanation of function can be best explained by using graphs, which makes functions reasonable.**

20. TRIGONOMETRY (SINE, COSINE AND AREA RULES

AREA RULE:

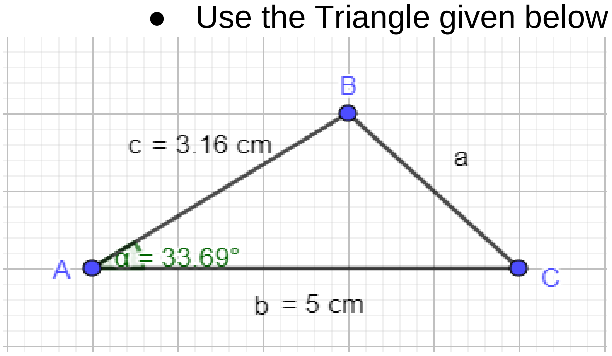


Area in $\Delta ABC = \frac{1}{2}bc\,sin\,sin\,A$

Area in $\Delta ABC = \frac{1}{2}ac\,sin\,sin\,B$

Area in $\Delta ABC = \frac{1}{2}ab\,sin\,sin\,C$

- We apply the **Area rule** if you are given the values of?
- Use the Triangle given below to find the Area and angles.



Answer:

Area in $\Delta ABC = \frac{1}{2} bc \ sin \ sin \ A$

Area in $\Delta\Delta ABC = \frac{1}{2}(5)(3.16) \ sin \ sin \ (33.69°)$

Area in $\Delta ABC = 4.38 \ cm$

Analysis:

Bloom taxonomy:

- Looking at this question the term ''**find''** is under ''**remember''** (first cognitive level) looking at the verbs in Blooms Taxonomy.
- Which means that, learners are expected to recall the basic information about triangle and its area
- In answering the question, **remembering** is the cognitive level expected.

Deductive reasoning:

- **General procedure is provided such that learner may use to reach a specific conclusion for instance the learners are provided with general rules like area rule (**$Area = \frac{1}{2} bc \ sin \ sin \ A$ **, by using them exact conclusion is expected which is the answer, an area of that triangle.**

Inductive reasoning:

- **Would have been using the example (which is in a form of a question in this case) to come up with the general rule, which is moving from specific examples to general conclusion.**
- **Link:** area rule grade 12 - YouTube

Rubric

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APPENDIX

LECTURER REVIEWS/COMMENTS

1. THREE QUESTIONS DEVELOP WHOSE ARE THOSE?
2. WHERE ARE THE REST OF QUESTIONS FOR THE LECTURER RECORD?
3. WHERE IS THE LESSON PLAN SIMILAR TO THE ONE, I USED AS A GUIDE, TO DEVELOP AND DISCUSS EACH LESSON, ATLEAST PART OF THAT SHOULD HAVE BEEN DEVELOPED? (31/AUGUST/2021)

4. DESIGN AN APPROPRIATE RUBRIC OR MEMO FOR THIS LESSON
5. IN ORDER FASHION AND AS ONE PIECE OF DOCUMENT – RETURN COMPLETE WORK AND THE RUBRIC ON 8TH SEPTEMBER. (03/09/2021)
6. GOOD WORK SO FAR BUT YOU NEED TO EXPAND EXACTLY WHAT NEEDS TO BE HERE IN HYPER LINKS?
7. LOOK FOR RESEARCH ARTICLES RELATED TO LESSON OUTCOMES
8. PUT TOGETHER ALL MY REVIEW COMMENTS AS PART OF EVIDENCE (19/09/2021)